2.2 Analyze Conditional Statements

Conditional statements are logical statements with two parts. When written in the *if, then* form, the "if" part contains the hypothesis and the "then" part contains the conclusion.

If p, then q. p-hypothesis q-conclusion

p→q "if p, then q" or "p implies q"

Examples:

If Cinderella completes her chores, then she can go to the ball.

If an angle is a right angle, then it measures 90

If a polygon has exactly 6 sides, then it is a hexagon.

Hypothesis Conclusion

Examples:

All squares are rectangles.

If it is a square, then it is a rectangle.

All cats are animals.

If it is a cat, then it is an animal.

Related conditionals

Negation-opposite meaning as well as opposite truth value (Symbol ~)

Conditional	p→q	If p, then q
Converse	q→p	If q, then p.
Inverse	~p→~q	If not p, then not q.
Contrapositive	~q→~p	If not q, then not p.

If a conditional is true, then the contrapositive must also be true. They are said to be logically equivalent. The same is true for the converse and the inverse.

Example: Put the following statement into the if, then form. Write each related conditional. Determine whether it is True or False. If false, provide a counterexample.

Example:
All birds are owls.
Conditional: False cardinal
If it is a bird, then it is an owl.

Converse: True
If it is an owl, then it is a bird.

Inverse: True
If it is not a bird, then it is not an owl.

Contrapositive:
If it is not an owl, then it is not a bird.
False Cardinal

Example: Write each related conditional. Determine whether it is True or False. If false, provide a counterexample.

If two angles form a linear pair, then they are adjacent angles. T

Converse: If 2 ∠s are adj. ∠s, then they form a lin. pair. F

Inverse: If 2 ∠s do not form a lin. pair then they are not adj. ∠s. F

Contrapositive: If 2 ∠s are not adj ∠s then they do not form a lin. pair. T

Example:
All whales are mammals.
Conditional:

Converse:

Inverse:

Contrapositive:

Biconditional Statement--conjunction of a conditional and its converse

iff "If and only if"

Example:
~~If~~ a quadrilateral has 4 right angles, ~~then~~ it is a rectangle.
If a quadrilateral is a rectangle, then it has 4 right angles

A quadrilateral has 4 right angles iff it is a rectangle.

2.3 Apply Deductive Reasoning

Deductive Reasoning--use facts, rules, definitions, properties, or the laws of logic to reach logical conclusions

Example:
If 2 angles are vertical, then they are congruent.

a. Given: $\angle 1$ and $\angle 2$ are vertical

Conclusion: $\angle 1 \cong \angle 2$ (valid)

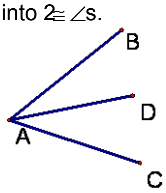
b. Given: $\angle 1 \cong \angle 2$

Conclusion: No Valid concl.

Example:
If a ray bisects an angle, then it divides it into 2 $\cong \angle$s.

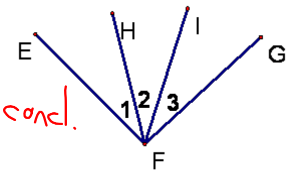
a. Given: $\overrightarrow{AD}$ bisects $\angle BAC$

Conclusion: $\angle BAD \cong \angle DAC$



b. Given: $\angle 1 \cong \angle 3$

Conclusion: no valid concl.



Example:
If a figure is a rectangle, then opposite sides are congruent.

a. Given: ABCD is a rectangle
Conclusion:

b. Given: MNOP is a valid trapezoid
Conclusion:

c. Given: Figure RSTU; RS = TU, ST = RU
Conclusion:

Examples of Law of Detachment

Law of Detachment--If p→q is true, and p is true then q is true.

Different type of reasoning.

Example:
Given: $\overline{WX} \cong \overline{UV}$; $\overline{UV} \cong \overline{RT}$

Conclusion: $\overline{WX} \cong \overline{RT}$

Example:
(1) If Casey gets to bat, then ~~he will~~ get a hit.

(2) If ~~Casey gets~~ a hit, then we will win the game.

(3) If Casey gets to bat, then we will win the game.

These are examples of the <u>Law of Syllogism</u>

If $p \rightarrow q$, and $q \rightarrow r$ are true, then $p \rightarrow r$ is true.

HW
p82-83
#s 1, 7, 11-15, 19, 20
p90
#s 4, 5, 7, 9, 12, 13, 15